INTERNATIONAL MATHEMATICS
TOURNAMENT OF TOWNS

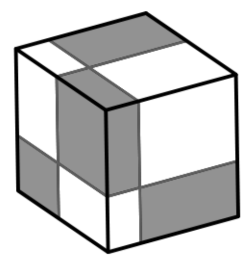
JUNIOR PAPER: YEARS 7, 8, 9, 10

Tournament 43, Northern Fall 2021 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. The Tournament of Towns is held once per year. This time the year of its (Northern) autumn round is divisible by the number of the tournament: $2021 : 43 = 47$. How many times more will humanity witness such a wonderful event? (4 points)

2. A cube is split into 8 boxes by three planes parallel to its faces. The resulting parts are painted in a chessboard pattern. The volumes of the black boxes are 1, 6, 8, 12. Find the volumes of the white boxes. (5 points)



3. A pirate has five purses with 30 coins in each. He knows that one purse contains only gold coins, another one contains only silver coins, the third one contains only bronze coins, and the remaining two purses contain 10 gold, 10 silver and 10 bronze coins each. It is allowed to simultaneously take any number of coins (including zero) out of each purse (only once), and examine them. What is the least number of coins which must be taken to determine for sure the content of at least one purse? (5 points)

4. A convex n-gon with $n > 4$ is such that any triangle formed by three consecutive vertices is always isosceles. Prove that there are at least 2 equal sides among any 4 sides of the n-gon. (5 points)

5. There were 20 participants in a chess tournament. Each of them played each other twice: once as white and once as black. Let us say that participant X is no weaker than participant Y if X has won at least the same number of games playing white as Y and also has won at least the same number of games playing black as Y. Can we be sure that there exist two participants A and B such that A is not weaker than B? (5 points)